12.6 — Quadric Surfaces

Goal: Learn seven 3D surfaces

- Cylinders,
- Paraboloids (two types)
- Hyperboloids (two types)
- Cones
- Ellipsoids

Entry Task and Warm Up:

What are the names of these 2D curves?

1. $3x + 2y = 1$
2. $3x^2 - y = 4$
3. $\frac{x^2}{9} + \frac{y^2}{4} = 1$
4. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Cylinders:

Key Idea: If one variable is missing, the 2D curve extends in the direction of the missing variable.

Examples:

- $x^2 + y^2 = 1$ - circular cylinder

- $z = y^2$ - parabolic cylinder

- $z = \sin(y)$ - sinusoidal cylinder

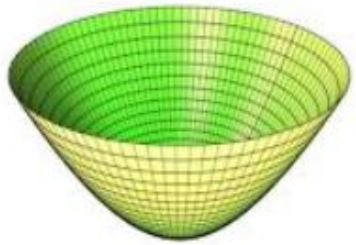
- $xy = 8$ - hyperbolic cylinder

Quadric Surfaces and Traces

Surfaces described by equations involving first- and second-degree terms in (x,y,z).

How do we visualize a surface?

Use **traces**: Fix one variable and examine the resulting 2D curve.



Elliptical/Circular Paraboloid

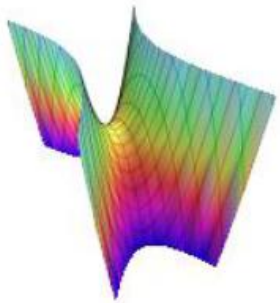
$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Examples *(only two variables square)*: HW Q11-12

- $z = 3x^2 + 5y^2$

- $y = 2x^2 - 5z^2$

- $x = z^2 - 2z + y^2 - 10y + 2$

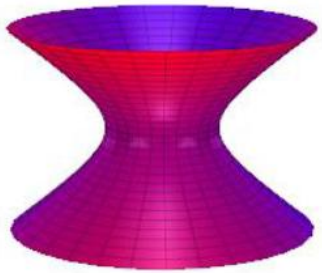


Hyperbolic Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

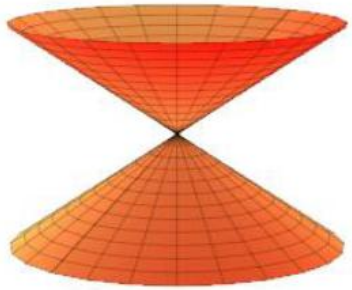
Ellipsoids, Cones, Hyperboloids *HW Q7–9*

When all three variables are squared…



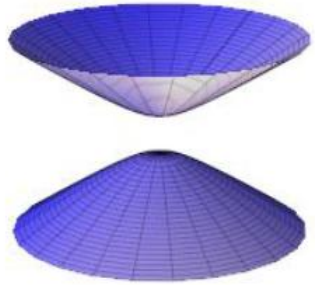
Hyperboloid of One Sheet

$$x^2 + y^2 - z^2 = 1$$



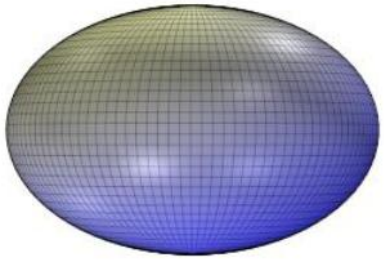
Cone

$$x^2 + y^2 - z^2 = 0$$



Hyperboloid of Two Sheets

$$x^2 + y^2 - z^2 = -1$$



Ellipsoid

$$x^2 + 2y^2 + 3z^2 = 12$$

Summary and Practice *HW Q1–16*

Strategy for recognizing a quadric surfaces:

1. Is a variable missing?

2. Are two variables squared?

3. Are all three variables squared?
   Examine signs and the constant.

4. Unsure? Take traces
   (really the best way, always do this!)

You do:

Identify several surfaces from equations on homework and old exams…